# Appendix for

# Simple Agreements for Future Equity (SAFEs): The not-so-simple search for simplicity

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In this Appendix we derive the formulas for calculating the main variables used across the four models. We only need to derive the mathematical expressions for the case of two angels. The case with one angel is a special case where the investment amount of the second angel is simply set to zero. Note also that it is straightforward to extend the model to more than two angel investments, we leave this as an exercise for the more enthusiastic readers.

The solution of the base model with a priced round follows the standard logic of staged equity financing, as explained in our textbook in Section 4.1.5. We briefly restate the key steps here as a reference case for the three other models. Mathematically speaking, all of these models consist of solving a set of equations with 'unknown' (or 'endogenous') variables that are derived on the basis of 'known' (or 'exogenous') variables. We find the unknow variables by solving the set of equations. Note that the accompanying spreadsheet shows all known values (assumptions) in green.

Throughout our analysis, we hold the ownerships stake of the VC constant. Thus, the central focus of our analysis is how different types of SAFEs affect the relative stakes of founders and angels. This is different than holding the price of the VC round constant. In that case different SAFE structures generate different number of shares, and result in different VC post-money valuations. We argue that holding the post-money valuation constant is the appropriate method, effectively assuming that the VC's valuation (or willingness to pay) remains unaffected by whatever SAFE structures were used before their investment.

## 1.) Priced rounds

We denote the investments by angel 1, angel 2 and the VC of the qualifying round by $\mathrm{I_i}$ where $i = 1,2, Q$, the shares by $\mathrm{S_i}$, the price per share by $\mathrm{P_i}$. We denote the post-money valuation of the qualifying round by $\mathrm{V_{POST}}$. From there we obtain the pre-money valuation by $\mathrm{V_{PRE}} = \mathrm{V_{POST}} - \mathrm{I_Q}$. Finally, it is useful to define the total number of shares after round i by $\mathrm{T_i}$, so that $\mathrm{T_1} = \mathrm{S_0} + \mathrm{S_1}$ is after the first angel round, $\mathrm{T_2} = \mathrm{S_0} + \mathrm{S_1} + \mathrm{S_2}$ is after the second angel round, and $\mathrm{T_Q} = \mathrm{S_0} + \mathrm{S_1} + \mathrm{S_2} + \mathrm{S_Q}$ after the VC round.

 

For a priced round, the known values are $S_0, I_1, I_2, I_Q, P_1, P_2,$ and $V_{POST}$ (and thus also $V_{POST}$. The unknown values are $S_1, S_2, S_Q,$ and $P_Q$. The key equations that need to be solved are:

$$I_1 = P_1 * S_1;\ I_2 = P_2 * S_2 \text{ and } I_Q = P_Q * S_Q$$

as well as:

$$V_{POST} = P_Q * T_Q, \text{ where } T_Q = S_0 + S_1 + S_2 + S_Q.$$

We use $S_1 = \frac{I_1}{P_1}$ and $S_2 = \frac{I_2}{P_2}$. This immediately expresses the first two unknown values in terms of known values. Next, we use these expressions in $T_2 = S_0 + S_1 + S_2$ to get:

$$T_2 = S_0 + \frac{I_1}{P_1} + \frac{I_2}{P_2}$$

which is now purely a function of known values.

We then use $T_Q = T_2 + S_Q$ in $V_{POST} = P_Q * T_Q$ to obtain:

$$V_{POST} = P_Q * T_2 + P_Q * S_Q = P_Q * T_2 + I_Q.$$

The pre-money valuation is defined as $V_{PRE} = V_{POST} - I_Q$. This only depends on known values. Rearranging the above expression, we obtain:[1]

$$V_{PRE} = V_{POST} - I_Q = P_Q * T_2$$

We thus obtain:

$$P_Q = \frac{V_{PRE}}{T_2}$$

This is an expression for the third unknown value $P_Q$, purely as a function of known values. The final unknown value is $S_Q$, which can now be obtained from:

$$S_Q = \frac{I_Q}{P_Q}$$

With all the unknown values solved, we can also calculate the total shares $T_r$ for $r = 1,2,Q$.

[1] This pre-money valuation is based on $T_2$ which includes all prior angel shares, i.e., $S_1$and $S_2$. Relative to the SAFE models, this pre-money valuation corresponds to the pre-money-post-SAFE valuation, defined as $V_{PRE}^{POSTSAFE} = P_Q * T_2$.

### 2.) SAFEs with discounts

The known values are $S_0, I_1, I_2, I_Q, P_1, P_2$, and $V_{POST}$. The unknown values are $S_1, S_2, S_Q$, and $P_Q$. The key equations to be solved are:

$$I_1 = P_1 * S_1 \text{ where } P_1 = (1 - D_1)P_Q$$
$$I_2 = P_2 * S_2 \text{ where } P_2 = (1 - D_2)P_Q$$
$$I_Q = P_Q * S_Q$$
$$V_{POST} = P_Q * T_Q,$$

It is useful to introduce the two types of pre-money valuation. First, there is a pre-money valuation that only focuses on the founders and excludes all angels who hold SAFEs. We call it the pre-money-pre-SAFE valuation and define it as:

$$V_{PRE}^{PRESAFE} = P_Q * S_0$$

Second, there is a pre-money valuation includes all prior shareholders, including all angels who hold SAFEs. We call this the pre-money-post-SAFE valuation and define it as:

$$V_{PRE}^{POSTSAFE} = P_Q * T_2$$

This corresponds to the pre-money valuation we derived for the case of a priced round, so unless there is any risk of confusion, we write $V_{PRE} = V_{PRE}^{POSTSAFE} = V_{POST} - I_Q$ which is a known value.

We are interested in determining $P_Q$ and can $P_Q = \frac{V_{PRE}}{T_2}$ for that purpose. The key challenge is to express $T_2$ in terms of known values. Practitioners sometimes have difficult determining to total number of shares prior to the round because the following circularity. In order to know the number of angel shares ($S_1$ and $S_2$) we need to know the angel prices ($P_1$ and $P_2$). With a SAFE, these prices depend on the VC price in the qualifying round ($P_Q$), but this price depends on the total number of shares, which includes the unknown angel shares. To resolve this circularity problem, we derive a useful expression for the total number of shares, specifically on $T_2$.

Define $\Phi_1$ and $\Phi_2$ as the ownership stake of angel 1 and 2 in the pre-money valuation of the qualifying round.[2] This ownership stake measure how many shares the angels have relative to the total number of shares (before the VC investment). Therefore, we have $\Phi_1 = \frac{S_1}{T_2}$ and $\Phi_2 = \frac{S_2}{T_2}$. We can therefore rewrite

[2] In terms of the ownership notation developed at the end of this appendix, this corresponds to $\Phi_1 = F_1(2)$ and $\Phi_2 = F_2(2)$, but for now we prefer the easier notation.

$$T_2 = S_0 + S_1 + S_2 = S_0 + \Phi_1 T_2 + +\Phi_2 T_2 \Leftrightarrow T_2(1 - \Phi_1 - \Phi_2) = S_0$$

which gives us

$$T_2 = \frac{S_0}{1 - \Phi_1 - \Phi_2}$$

This equation solves the circularity problem described above. Specifically, we can express $T_2$ as a function of know variables. All we need is to express $\Phi_1$ and $\Phi_2$ in terms of known values. For this we note that $\Phi_1 = \frac{S_1}{T_2} = \frac{I_1}{P_1 T_2} = \frac{I_1}{(1-D_1)P_Q T_2}$. Using $P_Q T_2 = V_{PRE}$, we get

$$\Phi_1 = \frac{I_1}{(1 - D_1)V_{PRE}}$$

Intuitively, this says that the ownership of angel 1 at the pre-money stage of the qualifying round is given by the investment $I_1$, divided by the discounted pre-money valuation $(1 - D_1)V_{PRE}$.[3] Since $I_1$, $D_1$, and $V_{PRE}$ are all known values, $\Phi_1$becomes a known value.

Similarly using $\Phi_2 = \frac{S_2}{T_2} = \frac{I_2}{P_1 T_2} = \frac{I_2}{(1-D_2)P_Q T_2}$, we obtain

$$\Phi_2 = \frac{I_2}{(1 - D_2)V_{PRE}}$$

so that $\Phi_2$ becomes a known value. Thus, $T_2 = \frac{S_0}{(1-\Phi_1-\Phi_2)}$ also becomes a known value.

From $P_Q = \frac{V_{PRE}}{T_2}$, $P_Q$ becomes known too. With $P_1 = (1 - D_1)P_Q$ and $P_2 = (1 - D_2)P_Q$ they finally also become known values. Finally, using $S_1 = \frac{I_1}{P_1}$, $S_2 = \frac{I_2}{P_2}$, and $S_Q = \frac{I_Q}{P_Q}$ we obtain the remaining number of shares. We have thus solved the entire model.

### 3.) SAFEs with pre-money caps

For simplicity we only consider the case where both angel 1 and 2‘s caps are binding. In this case we have:

$$P_1 * S_0 = V_{CAP,1}^{PRE} \text{ and } P_2 * S_0 = V_{CAP,2}^{PRE}$$

The known values are $S_0, I_1, I_2, I_Q, V_{CAP,1}^{PRE}, V_{CAP,2}^{PRE}$, and $V_{POST}$. The unknown

[3] Alternatively, we can think of it as a discount-augmented investment $\frac{I_1}{(1-D_1)}$ divided by the pre-money valuation $V_{PRE}$.

variables are $S_1, S_2, S_Q, P_1, P_2, P_Q$. With a binding cap, and using the two equations above, we obtain

$$P_1 = \frac{V_{CAP,1}^{PRE}}{S_0} \text{ and } P_2 = \frac{V_{CAP,2}^{PRE}}{S_0},$$

so that $P_1$ and $P_2$ become known values. From here on, the structure of the model is identical to the model with a priced round, and we can simply use the formulas we already derived there.

## 4.) SAFEs with binding post-money caps

We will first focus on the case where the price caps for both angels 1 and 2 are binding. We can write the post-money cap as follows:

$$P_1 * T_2 = V_{CAP,1}^{POST} \text{ and } P_2 * T_2 = V_{CAP,2}^{POST}$$

The key difference to the pre-money cap is that now the capped price is multiplied with the total shares $T_2$, not just the founder shares $S_0$.

The known values in this model are $S_0, I_1, I_2, I_Q, V_{CAP,1}^{POST}, V_{CAP,2}^{POST}$, and $V_{POST}$. The unknown values are $S_1, S_2, S_Q, P_1, P_2$, and $P_Q$. The key equations to be solved are now:

$$I_1 = P_1 * S_1;\ I_2 = P_2 * S_2;\ I_Q = P_Q * S_Q; \text{ and } V_{POST} = P_Q * T_Q$$

Consider again the ownership in the pre-money valuation:

$$\Phi_1 = \frac{S_1}{T_2} = \frac{I_1}{P_1 * T_2} = \frac{I_1}{V_{CAP,1}^{POST}}$$

And similarly

$$\Phi_2 = \frac{S_2}{T_2} = \frac{I_2}{P_2 * T_2} = \frac{I_2}{V_{CAP,2}^{POST}}$$

These are simple expressions of the pre-money ownership stakes that only depend on known values. We immediately obtain $T_2 = \frac{S_0}{(1-\Phi_1-\Phi_2)}$ which is based on known values.

This allows us to express the unknown angel shares as:

$$S_1 = \Phi_1 * T_2 \text{ and } S_2 = \Phi_2 * T_2$$

which is a function of the known values. The prices paid by angels follow directly using $P_1 = \frac{I_1}{S_1}$ and $P_2 = \frac{I_2}{S_2}$.

Again, we use $V_{PRE}^{POSTSAFE} = P_Q * T_2$ to obtain:

$$P_Q = \frac{V_{PRE}^{POSTSAFE}}{T_2}$$

This is an expression for the unknown $P_Q$ as a function of known values. The final step is to find the VC shares using $S_Q = \frac{I_Q}{P_Q}$. We have thus expressed all unknown values in terms of the known values.

### 5.) SAFEs with partially binding post-money caps

Consider the case where there are two SAFE investments with different post-money caps, but only one of them is binding. The more typical case is when the second cap is higher than the first, so that $V_{CAP,1}^{POST} < V_{CAP,2}^{POST}$. Suppose now that $V_{CAP,1}^{POST} < V_{PRE} < V_{CAP,2}^{POST}$, so that the first but not the second cap is binding.

For the first angel with a binding cap, we obtain as before $\Phi_1 = \frac{S_1}{T_2} = \frac{I_1}{P_1 * T_2}$, so that

$$\Phi_1 = \frac{I_1}{V_{CAP,1}^{POST}}$$

For the second angel without a binding cap, we obtain the same ownership stake as with just a discounted price. Specifically, $\Phi_2 = \frac{S_2}{T_2} = \frac{I_2}{P_2 T_2} = \frac{I_2}{(1-D_2) P_Q T_2} = \frac{I_2}{(1-D_2) V_{PRE}}$, so that

$$\Phi_2 = \frac{I_2}{(1 - D_2) V_{PRE}}$$

From this we can use the same logic as before, i.e., calculate $T_2 = \frac{S_0}{(1-\Phi_1-\Phi_2)}$, and then $P_Q = \frac{V_{PRE}^{POSTSAFE}}{T_2}$ and then all the shares $S_1 = \frac{I_1}{P_1}$, $S_2 = \frac{I_2}{P_2}$, and $S_Q = \frac{I_Q}{P_Q}$.

In the rare case where the second cap is binding, but the first is not, we can again use the same approach, but simply switching the formulas, so that $\Phi_1 = \frac{I_1}{(1-D_1) V_{PRE}}$ and $\Phi_2 = \frac{I_2}{V_{CAP,2}^{POST}}$.

## 6.) Ownership stakes

We finally take a look at ownership fractions $\mathrm{F_i(r)}$. We will specifically focus on the (initial) angel investor (i.e., i=1) and two closely related ownership stakes. First, we will examine $\mathrm{F_1}(Q)$ which is the angel's ownership after the dilution of the qualifying round. This will be based on the post-money valuation of the qualifying round. Second, we will examine $\mathrm{F_1}(1)$ (if there is only one angel round) or $\mathrm{F_1}(2)$ (if there are two angel rounds). This is the initial angel's ownership before the dilution of the qualifying round and is based on the (post-SAFE-) pre-money valuation of the qualifying round.

For the remainder of this section, we only consider $\mathrm{V_{PRE}^{POSTSAFE}}$ and will revert to the shorter notation $\mathrm{V_{PRE}}$. In addition, we introduce notation from the pre- and post-money valuations of the first and second angel round, denoted by $\mathrm{V_{PRE,1}}$, $\mathrm{V_{POST,1}}$, $\mathrm{V_{PRE,2}}$, and $\mathrm{V_{POST,2}}$. We first derive some properties of the angel stake that can be applied across all the different types of securities:

$$\mathrm{F_1}(Q) = \frac{\mathrm{S_1}(Q)}{\mathrm{T}_Q} = \frac{\mathrm{P}_Q}{\mathrm{P_1}}\frac{\mathrm{P_1 S_1}(Q)}{\mathrm{P}_Q \mathrm{T}_Q} = \frac{\mathrm{P}_Q}{\mathrm{P_1}}\frac{\mathrm{I_1}}{\mathrm{V_{POST}}}$$

$$\mathrm{F_1}(2) = \frac{\mathrm{S_1}(2)}{\mathrm{T_2}} = \frac{\mathrm{P}_Q}{\mathrm{P_1}}\frac{\mathrm{P_1 S_1}(2)}{\mathrm{P}_Q \mathrm{T_2}} = \frac{\mathrm{P}_Q}{\mathrm{P_1}}\frac{\mathrm{I_1}}{\mathrm{V_{PRE}}}$$

We now define:

$$\Delta = \frac{\mathrm{P}_Q}{\mathrm{P_1}}$$

as the "price multiple" which measures the increase in the price paid by the angel to the price paid by the VC in the qualifying round. The variation across SAFEs can be understood in terms of their different price multiple. Specifically, all ownership stakes can be expressed as a function of the price multiple, namely:

$$\mathrm{F_1}(Q) = \Delta\frac{\mathrm{I_1}}{\mathrm{V_{POST}}} \text{ and } \mathrm{F_1}(2) = \Delta\frac{\mathrm{I_1}}{\mathrm{V_{PRE}}}$$

We can thus characterize ownership stakes solely in terms of their price multiples $\Delta$.

Consider first the benchmark case of a priced round. With only a single angel round we have:

$$\Delta_{\mathrm{Priced}} = \frac{\mathrm{P}_Q}{\mathrm{P_1}} = \frac{\mathrm{P}_Q \mathrm{T_1}}{\mathrm{P_1 T_1}} = \frac{\mathrm{V_{PRE}}}{\mathrm{V_{POST,1}}}$$

Intuitively, the price multiple measures how the shares appreciate from the first round (looking at the post-money valuation then which includes all round 1shareholders) to the qualifying round (looking at the pre-money valuation then which includes all round 1 shareholders). In the case of two angel rounds, this is given by:

$$\Delta_{\text{Priced}} = \frac{P_Q}{P_1} = \frac{P_2 T_1}{P_1 T_1}\frac{P_Q T_2}{P_2 T_2} = \frac{V_{\text{PRE},2}}{V_{\text{POST},1}}\frac{V_{\text{PRE}}}{V_{\text{POST},2}}$$

The intuition is the same, except that the price multiple has two components, one reflecting the value increase from the first angel to the second angel round, and one from the second angel round to the qualifying VC round.

Consider next the case of a SAFE with a discount. The price multiple is simply a function of the price discount. Specifically:

$$\Delta_{\text{Discount}} = \frac{P_Q}{P_1} = \frac{P_Q}{P_Q(1 - D_1)} = \frac{1}{1 - D_1}$$

This formula applies irrespective of whether there are one or two angel rounds. The intuition is simply that a higher discount generates a higher price multiple.

Consider next a binding pre-money cap. Consider first the case of a single angel round. It is useful to note that $P_1 T_1 = P_1 S_0 + P_1 S_1 = V_{\text{CAP},1}^{\text{PRE}} + I_1$. With this we note that:

$$\Delta_{\text{Precap}} = \frac{P_Q}{P_1} = \frac{P_Q T_1}{P_1 T_1} = \frac{V_{\text{PRE}}}{V_{\text{CAP},1}^{\text{PRE}} + I_1}.$$

This also implies a simple expression of the ownership fractions, namely:

$$F_1(Q) = \frac{I_1}{V_{\text{CAP},1}^{\text{PRE}} + I_1}\frac{V_{\text{PRE}}}{V_{\text{POST}}} \text{ and } F_1(2) = \frac{I_1}{V_{\text{CAP},1}^{\text{PRE}} + I_1}$$

For the case of two angel rounds, we use:

$$P_1 S_2 = \frac{P_1 S_0}{P_2 S_0} * P_2 S_2 = \frac{V_{\text{CAP},1}^{\text{PRE}}}{V_{\text{CAP},2}^{\text{PRE}}} * I_2$$

which we can use in:

$$P_1 T_2 = P_1 S_0 + P_1 S_1 + P_1 S_2 = V_{\text{CAP},1}^{\text{PRE}} + I_1 + \frac{V_{\text{CAP},1}^{\text{PRE}}}{V_{\text{CAP},2}^{\text{PRE}}} * I_2.$$

With this we derive:

$$\Delta_{\mathrm{Precap}} = \frac{\mathrm{P}_Q}{\mathrm{P}_1} = \frac{\mathrm{P}_Q \mathrm{T}_2}{\mathrm{P}_1 \mathrm{T}_2} = \frac{\mathrm{V}_{\mathrm{PRE}}}{\mathrm{V}_{\mathrm{CAP,1}}^{\mathrm{PRE}} + \mathrm{I}_1 + \frac{\mathrm{V}_{\mathrm{CAP,1}}^{\mathrm{PRE}}}{\mathrm{V}_{\mathrm{CAP,2}}^{\mathrm{PRE}}} * \mathrm{I}_2}.$$

Finally, consider a binding post-money cap. We have:

$$\Delta_{\mathrm{Postcap}} = \frac{\mathrm{P}_Q}{\mathrm{P}_1} = \frac{\mathrm{P}_Q \mathrm{T}_2}{\mathrm{P}_1 \mathrm{T}_2} = \frac{\mathrm{V}_{\mathrm{PRE}}}{\mathrm{V}_{\mathrm{CAP,1}}^{\mathrm{POST}}}.$$

This formula applies equally to the case of one or two angel rounds. Moreover, it implies:

$$\mathrm{F}_1(Q) = \frac{\mathrm{I}_1}{\mathrm{V}_{\mathrm{CAP,1}}^{\mathrm{POST}}} \frac{\mathrm{V}_{\mathrm{PRE}}}{\mathrm{V}_{\mathrm{POST}}} \text{ and } \mathrm{F}_1(2) = \frac{\mathrm{I}_1}{\mathrm{V}_{\mathrm{CAP,1}}^{\mathrm{POST}}}$$

This gives an extremely simple formula for determining the ownership of the angel in the qualifying round. Specifically, the post-money cap determines the pre-money valuations that the angel investment gets in the qualifying round.